The Laws and Rules of Business
Theory and Practice

Jon Michael Smith

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Abstract

Some American businesses succeed, while many others fail: why? Many have opinions about why. Whether we are talking case histories, media reports, court cases, or the oral tradition of the great business leaders in their autobiographical books, all seem flawed because the document successes and failures without the benefit of fundamental insight. Different than simple success formulas, but important questions to answer are, “Are there natural laws for business, which can help understand and explain the successes and failures experienced by businesses in America?” “What does modern information systems theory and systems simulations have to say about monitoring businesses so as to avoid failure?” “What is it about accounting systems that constrain their ability to help guide business away from failure and toward success?” Many agree that we need to find new ways to understand business failures and successes in America if she is to maintain business leadership in the world.

At first, the answers seemed to indicate there would be no help from these points of view. Business is simply too complex to capture in a simple set of rules, natural laws or simulations. However, by reducing “business” to an assembly of profit-cells, each cell of which MUST make a profit, there are fundamental laws, which these cells must satisfy if the cell is to be profitable. Violating these laws leads to failure.

Examining these laws lets us distill the essence of business into a crisp, short set of commonsense rules that have a solid foundation in analysis and empirical test. Some of these rules seem counter-intuitive at first. On reflection, they will be reliable replacements for misconceptions about business that often plague the new entrants to business1.

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[1] [1]Mr. Smith is employed by Mary Washington College and by the National Aeronautics and Space Administration (NASA). He is an adjunct professor in the department of Business Administration where he teaches Quantitative Methods of Analysis in business. He is a program manger in NASA’s Office of Advanced Concepts and Technology where he manages the Advanced Communications Technology Satellite Program. Mr. Smith is a member of the Harvard business School 84th Advanced Management Program. The opinions presented in this paper are entirely his own and don not represent the opinions of either employers.

1 Introduction

Some say that doing business is an art; an intuitive skill forged in the crucible of competition. Others say that business is like a game, with rulebooks and playbooks and where profit is the score. Some even hold that business is like war. Like war, business can be mathematically modeled, gamed, and played at business colleges in a manner similar to the way war games are played at war colleges, military academies and in operational military commands using battlefield simulators and fight simulators. Whatever the point of view, it appears that there are fundamental laws that underlie the doing of profitable business. Whether real or simulated, guided by intuition or by analysis, every business must obey these laws if it is to survive. For sure, businesses that fail violate these laws. In this sense, avoiding failure ensures success. However this work strikes the reader, this paper is about the laws of business and the reliable, common sense rules that flow from them. These rules, when combined with the businesspersons' intuition are a more powerful weapon for competing in the market place, than either one alone.

For the business manager, information is power. Mathematical models are useful because they contain so much information at very little cost. In recent years, M. J. Feigenbaum[2] has shown the benefits of employing the simplest model of non-linear dynamic systems as the basis for systems analysis and simulation. His extraordinary contribution was finding universal properties in the chaotic behavior in many non-linear systems. In what follows here, we will take the view that simpler is better when modeling the nonlinear behavior of business profit. Our focus on profit stems from the notion that business must be profitable to survive. Said differently, avoiding failure is a way to succeed. The work here has lead to the development of business caution and warning systems to aid the business manager in avoiding failure as one of the tools that will lead to business success.

Mathematical modeling is deceptively easy to do. Together we will write the simplest, most general mathematical model for the profit generated from a small element (cell) of business. This formula will include only the fewest variables that it takes to model the profit. We will analyze the formula to determine what rules we might find that apply to the business cell, which we can scale-up to full-size business operations[3].

We will deduce from these laws the practical leveraged rules that accelerate progress along a path toward continually improving profits and success. We can expect that the combination of the mathematical analysis of business and the manager's business intuition, when used together, will be better than either one alone. Let us begin, then, with the definition of profit.

2 The Definition of Profit

Profit is defined to be the difference between revenue and cost. Revenue is defined to be the product of the price and the number of

2 Feigenbaum, MJ: Quantitative Universality for a Class of Nonlinear Transformations, J. Stat. Phys. 19, 25. 1978

3 The fundamental laws of business show the quantitative relationship among the fewest variables in business. The application of the laws in a business simulation may involve hundreds of formulas that account for the many interacting cells of a business that produce a profit or result in a loss.

sales. The symbol for price is P and the number of sales[4] in a given period by N_S. We can write a formula for revenue of a single source of cell of profit as follows:

$$R = PN_S$$

Cost is defined to be the sum of the direct cost of sales and the total fixed cost of operating the business for the same period of time as the revenues. The total direct cost is defined to be the product of the cost of the item sold and the number of sales over a given period. The total fixed cost is defined to be the product of the periodic fixed-cost of doing business and the units of time over the same period as the direct costs are accumulating. The time basis can be a week, a month, a year, the life cycle of the product or any basis the analyst chooses to use. What is important is that the number of sales and the number produced are on the same time basis as the fixed cost.

In all businesses the total fixed costs are those costs that are independent of sales but necessary for the conduct of business and continue in time, whether any product/service is sold or not. These costs include rent, insurance, utilities, equipment's, management and staff, accounting and legal services etc., and are relatively fixed when compared with the variable costs of selling. We denote the "fixed" cost with the symbol F and time with the symbol T[5].

Now, the total cost of doing business is the total direct cost and the total fixed cost over the period of interest. Typically, a business investment horizon is typically a year, and business performance is measured annually and the numbers of products sold and produced are annual sales and annual production.

We will denote the total fixed cost[6] with the symbol FT : here T denotes the number of periods in the analysis. We will denote the second term with the product VN_p where V is the unit cost[7] of the item sold and where N_p is the number of items produced or procured in a year. We can write the cost formula as follows[8]:

$$C = VN_p + FT$$

We call V the unit variable cost. We call the product VN_p the total "variable" cost (because it varies with the number of items produced or procured). In the same way that revenue depends on the number of sales, the variable-cost depends on the number of units

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[4] In this paper we refer to "products" but the analysis applies equally as well to services. Read "products" as "products or services or both".

[5] The variables *P,V,F&T* can be in any dimensions provided they are consistent among themselves.

[6] In this paper, F is the periodic fixed operating cost. If no products are sold, there is still the periodic cost of rent, utilities, staff, etc., and these are fixed periodic costs that aggregate to a total fixed cost. In our model, the cost of advertising will be included in the fixed costs.

[7] The value of the product V is the coat to produce it, and N_p is the number of units produced. The cost V includes the cost of labor, materials, and equipment used in the fabrication of the product. In this sense it is the economic value of the product, hence the term V.

[8] The symbols used in this paper are not the same as those used in the caution and warning system FIRST WARNING. They are nearly the same. The translation is simply $V_u = V, F_m = F$. In FIRST WARNING V_u is the unit variable cost and F_m is the monthly fixed costs (bills) for operating the business.

produced or procured. Substituting the revenue and cost formulas into the definition of profit we find:

$$\text{Profit} = P_r = PN_s - VN_p - FT$$

Let us assume[9] that $N_p = N_s = N$. Now we can write the profit definition in the form:

$$P_r = (P - V)N - FT$$

Traditional break-even analysis shows a lot about business at this point. This formula shows us that to be profitable the first term in this formula must be positive and be greater than the second term. For the first term to be positive, *P* must be greater than *V*. The Price of the item sold must exceed the unit-value of the item. Provided this is true, the contribution to the profit from the first term will eventually exceed the total fixed costs and the business will become profitable. The point where the contribution to profit exactly equals the total fixed cost is called the break-even number of sales. The break even is calculated with the formula

$$N_{breakeven} = \frac{FT}{P - V}$$

To be profitable, a business must satisfy two conditions. The first is that the price must exceed the unit-value of the product being brought to market. The second is that the volume of sales will produce enough contribution to the profit to exceed the fixed cost.

3 Scale-up

Envision a spreadsheet program where an array of profits from each product offered by a business is:

$$\begin{aligned} P_{r1} &= (P_1 - V_1) - F_1 T, \\ P_{r2} &= (P_2 - V_2) - F_2 T, \\ P_{r3} &= (P_3 - V_3) - F_3 T. \\ &\vdots \qquad\qquad \vdots \end{aligned}$$

One can either:

- Build a spreadsheet by using a column for the *Profit*, a column for *P*, a column for *V*, for *N* and for *F*. Note that *T* is generally fixed for the time period being analyzed, or
- Build the spreadsheet using a single column with *P*, *V*, *N&F* read from a data table, or

[9] In this model, we will assume that the number of sales (volume) is the same as the number produced. Note: To focus on the essentials of business, we make simplifying assumptions. Simplifying extracts the essentials of business from the untidy affairs of the real world. Such is the tremendous benefit of analysis. The chaos of the real world can be extraordinarily distracting and time consuming leaving little time or energy for understanding the essence of business.

- Use a Monte Carlo simulation of the many products and services and their interaction to optimize the profitability of the business.

Whatever way, ensemble sums, averages, standard deviations and other arithmetic and statistical functions can be used to determine the profitability of the business.

A spread sheet arranged to show a forecast of monthly profits for different products by row (product) and by column (month), the matrix of profits can be analyzed as either a time series (across the columns) or as an ensemble array (down each column) and the profitability and associated profit and breakeven forecast confidence & associated risks for the entire business can be determined to aide in the management of the business. Preparing spreadsheets in this manner avoids the common mistake of inferring from the time series statistics what the ensemble statistics of a business will be, and visa-versa.

We see then that the profitability of the business, in total can be developed with spreadsheet analysis. We can also see now that each cell of profit must satisfy the laws of business thus far developed if the profitability of the business on the whole is to be maximized. In this sense, the laws of business for the individual cell must be well understood in order that the ensemble performance of the business can be well understood. Additionally, once the spreadsheet is developed, the sensitivity and leverage associated with each parameter in the analysis can be determined and provide guidance on how to improve profitability in a practical manner. Leverage is discussed in further later in this paper. For now, we will focus on the laws of profitability as related to the individual cell, inferring that what applies to the profit cell, applies to business profitability in the large.

4 The Laws of Supply and Demand

While useful for some discussions of business, the profit and break-even concepts studied in the previous sections treat the sales volume as independent of the price. We all know, however, that they are not independent of one another. The relationship between the two is called demand elasticity and is much studied by economists. The demand law[10] we will use here is the same demand law found in studies of microeconomics. The profit formula by itself is a definition. From the definition, we can learn about the relationships between the price, the unit value of the product and the number of products that have to be sold to break-even. What else can we learn? The answer lies in the same profit formula, modified to include the natural law of demand.

The variables in the profit definition (formula) are not functionally related: *N&P* are not related. We know, however, that the higher the price, the less the demand and the fewer the sales. Some say that the demand is elastic, and declines as price increases. The mathematical model for this relationship, in simplest terms, is nonlinear[11] and takes the form

[10] Economists study the micro-economic laws of supply and demand, in part, to establish the value of goods and services in a fair market economy.

[11] The relationship between demand and price is typically a nonlinear function. The non- linear demand steps down as price thresholds are exceeded. This type of nonlinearity (a stairstep nonlinearity relating demand to price) can be approximated with a polynomial that fits the average of each step. The polynomial can then be Taylor Series expanded, the first two terms of which are linear. This way of thinking about modeling price elasticity is even simpler than $N = N_{max}(1 - P/P_{max})^n$, model we discuss in this paper and further justifies using the n=1 case for analysis in this paper.

$$N = N_{\max}\left(1 - \frac{P}{P_{\max}}\right)^n$$

where n[12] is a rational number selected empirically to fit the shape of the price elasticity curve. Where such information is unavailable, a reasonable value for n is 1 for many businesses. We will use the $n = 1$ case for our analysis in this paper as the results are similar for all other cases. Where appropriate, comments will be added when the specific $n = 1$ case and the more general n = rational number case differ. Then

$$N = N_{\max}\left(1 - \frac{P}{P_{\max}}\right)$$

This can also be written in the form,

$$N = \frac{N_{\max}}{P_{\max}}\left(P_{\max} - P\right)$$

These relationships show that when the price is zero (a give-away), the number of sales is at the maximum N_{max}. On the other hand, if the price is at the maximum marketable price P_{max}, the number of sales is zero. This relationship describes the behavior of consumers in a free-market economy.

Now the demand should be met by the supply of services and products from the business. There are many examples of businesses that under priced their products and services and were simply overwhelmed by the demand that the business could not meet. Peoples Express Airline is a classic example taught at MIT Sloan Business School. The bottom line is that inexperience business managers can get into trouble FAST by not considering the need to meet the demand in the business.

The ability to supply the demands for products by the business is called the capacity of the business to meet the demand, Nc. While Nmax and Pmax and n characterize the market demand, Nc characterizes the businesses ability to meet the demand. If there are four numbers that are needed to enter the business world in a professional manner, it is these four numbers. Estimate them, guess them, or do a market study to determine them. However, you do it, get them in your mind and study them, as they are not part of many business plans. The three numbers associated with the behavior of consumers in the market place are critical for sure. So to is the capacity of the business that must supply the demand. Studies of business failures show conclusively that businesses fail in the market place by not staying on top of market trends and capacity planning. However you do it, get a handle on these four numbers and their implication to the size and staffing of the business.

5 The Laws of Business

When we combine the profit definition and the demand law we find a set of three formulas that relate to profit. These formulas are called laws in the sense that they show how business profits are related to price

[12] In FIRST WARNING, the term for n is e so as not to confuse elasticity with sales.

through demand behavior in a free market. Substituting the natural law of demand elasticity into the revenue, cost, and profit formulas (definitions) we find:

$$Revenue = PN_{\max}\left(1-\frac{P}{P_{\max}}\right)^{n}$$

$$Cost = VN_{\max}\left(1-\frac{P}{P_{\max}}\right)^{n} + FT$$

$$Profit = (P-V)N_{\max}\left(1-\frac{P}{P_{\max}}\right)^{n} - FT$$

For the simpler n=1 case, we find:

$$Revenue = PN_{\max}\left(1-\frac{P}{P_{\max}}\right)$$

$$Cost = VN_{\max}\left(1-\frac{P}{P_{\max}}\right) + FT$$

$$Profit = (P-V)N_{\max}\left(1-\frac{P}{P_{\max}}\right) - FT$$

Note that this set of formulas can be written in the following slightly different forms:

$$Revenue = \frac{PN_{\max}}{P_{\max}}\left(P_{\max}-P\right)$$

$$Cost = \frac{VN_{\max}}{P_{\max}}\left(P_{\max}-P\right) + FT$$

$$Profit = \frac{N_{\max}}{P_{\max}}\left(P_{\max}-P\right)\left(P-V\right) - FT$$

We will have occasion to use both forms in this paper.

These formulas say much about what is required to operate a small element of a profitable business. Among the more interesting are:

- There is a price for every item that will maximize profit. This can be seen by noting (in the second form of the profit equation) that profit is polynomial in price. The roots of the polynomial are V and at $P = P$max the profit is $-FT$. Thus as the price varies from V to P_{max}, the profit rises from $-FT$ to a peak value then drops down again to *-FT*. Obviously, the peak profit occurs when the price is half way between *V* and P_{max} as

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$$P_{optimum} = \frac{V + P_{\max}}{2}$$

The more general result is similar and takes the form:

$$P_{optimum} = \frac{nV + P_{\max}}{n+1}$$

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- There is a price for every item that will maximize revenue. This can be seen by noting (again in the second form of the *n* = 1 revenue equation) that revenue is also a polynomial in price. The roots of the polynomial are 0 and P_{max}. At *P* = 0 the revenue is 0. Also at P = Pmax the revenue is also 0. Thus as the price varies from 0 to P_{max}, the revenue rises form 0 to a peak value then drops down again to 0. Obviously, the peak revenue occurs when the price is half way between 0 and P_{max} as
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$$R_{optimum} = \frac{P_{\max}}{2}$$

- The price that maximizes profit is different from the price that maximizes revenues. The price that maximizes revenue is less than the price required to maximize profit and thus under-prices the business and profit is lost.

- The maximum profit can be predicted. The predicted maximum profit can be used as a metric to assess the performance of actual business operations, and comparing the predicted maximum profit with actual business performance can be used to decide when a business has reached its peak and it is time to change growth strategies for the business.

A further analysis of the messages in these formulae will be given in the section entitled *An Analysis of Business Laws*.

6 Practical Considerations

Perhaps the most practical consideration that comes from this analysis that the profit law shows what there are only eight variables are of fundamental importance for making a profit. *It tells the management that while there are many variables that require attention, there are only eight FUNDAMENTAL variables that will make or break the business: monitoring the others amounts to determining their effect on these eight variables. The eight variables are;*

1. *The sales volume were the price = 0 (free of charge),* N_{max}
2. *The price where the sales volume drops to zero,* P_{max}
3. *The elasticity of the sales price relationship, n*
4. *Actual selling Price, P*
5. *The number of units to be sold, N*
6. *Unit Variable-Cost (value) of the product, V*
7. *Total Fixed Cost, FT*
8. *Capacity of the business to meet the demands of the market, Nc*

Whether on an individual product basis or in an aggregated average over all products offered by the business, here is where the focus should be if management is to succeed. Profit, success, and survival will follow from an understanding of these fundamental variables. Said differently, failure in a business to become a profitable, financially self-sustaining operation can be attributed to a misunderstanding of one or more of these variables and their interrelationships as established in the profit laws developed here: Nothing is more fundamental.

The second consideration is that *the management of a business should require an accounting by their subordinate managers of the prices they set and the degree to which they contribute to the profitability of a business.* The VP for Marketing should be accountable for revenues. The VP for cost should be accountable for the unit variable cost. The VP for finance should be accountable for periodic fixed cost, interest and taxes.

The third consideration derives from the determination that the price that maximizing revenue does not maximize profit. *If Managers focus on maximizing revenues, and then the business will be sub optimized in profit.* More will be developed on this matter, later in this paper.

7 An Analysis of the Profit Law

The n = 1 profit formula involves many terms with the seven of the eight fundamental variables embedded in each of the terms. To understand the effect of each variable it is necessary to isolate the variable so that it only effects a single term in the profit formula. Fortunately, algebra makes this easy to do. The benefit is that clear and correct conclusions can be made about the cause and effect relationship has between each variable and profit.

The n = 1 profit formula can be written in six different algebraic forms. Each form isolates a different fundamental parameter so that it is not combined with other parameters. In this way we can see how each of the fundamental variable effect profit. The variables that effect business are shown in brackets bold font. The six forms are as follows:

$$Profit = -VN_{max}\left(1-\frac{P}{P_{max}}\right)+N_{max}\left(P-\frac{P^2}{P_{max}}\right)-FT \qquad (1)$$

$$Profit = -\frac{N_{max}}{P_{max}}P\left[P-V\right]+N_{max}\left(P-V\right)-FT \qquad (2)$$

$$Profit = +N_{max}\left[\left(P-V\right)+V\left(\frac{P}{P_{max}}\right)-\frac{P^2}{P_{max}}\right]-FT \qquad (3)$$

$$Profit = -FT+N_{max}\left[\left(P-V\right)+V\left(\frac{P}{P_{max}}\right)-\frac{P^2}{P_{max}}\right] \qquad (4)$$

$$Profit = +\left[\frac{N_{\max}}{P_{\max}}\right]P^2 + \left[N_{\max}\left(\frac{V}{P_{\max}}\right)-1\right]P - \left[N_{\max}V + FT\right] \qquad (5)$$

$$Profit = +(P-V)(P_{\max}-P)^n \frac{N_{\max}}{P_{\max}} - FT \qquad (6)$$

What follows is a variable-by-variable comment of the relationship between profit and the five variables of business operations. Because care has been taken to isolate the influence of these variables on profit, these findings can be a confident basis for intuitive business operations. In a sense, they verify what intuition might guide a businessperson to do. What is unique about this analysis is that it focuses on only five fundamental variables and results in a deep understanding of the non-linear relationship between profit and a products sales price.

- The first form shows that the negative sign associated with the unit variable-cost *V*, is to reduce profit, an intuitive result that guides us to the common sense rule that profit w always improved by decreasing the unit variable-cost. Wherever possible, reduce the unit variable-cost of the products sold.
- The second form shows that provided the price exceeds the unit variable-cost, the first term is negative and thus subtracts form profits. To make this term as small as possible, we see the profit is always improved by selling products that command a large maximum marketable price P_{max}.
- The third form shows that profit is always improved by selling product that commands a large maximum sales volume N_{max}.
- The fourth form shows that profits are reduced by the total fixed cost *FT* of the business. If the time period in the analysis is a year, *F* is the fixed annual cost and *T* is the number of years being considered in the analysis. If the analysis is a life cycle cost, *T* is the number of years in the life cycle. Whatever the time basis of the analysis, this form argues for keeping the total fixed cost as low as possible.
- The fifth form of the profit law shows that profit is a quadratic function of price *P*. As mentioned before, Profit as a function of price is a parabola with a maximum profit when

$$P_{\text{optimum}} = \frac{V + P_{\max}}{2}$$

- The sixth form of the profit law shows that the elasticity parameter *n* STRONGLY influences profit. We can see here that as *n* varies over the small range from zero to three, the profit varies dramatically from a constant inelastic demand, independent of price to a briskly declining highly elastic demand that is highly price sensitive. In the limit, as n approaches ∞, the demand approaches zero. It is important to carefully assess this parameter. A relationship that can be used to estimate its value, based on historical data is as follows. To use this relationship, it is suggested that a number of samples of *N* be taken for values of *P* and an average *n* be used for analysis.

$$n = \frac{Ln(N / N_{\max})}{Ln(1 - P / P_{\max})}$$

8 Pricing for Maximum Profit

A necessary condition to achieve the maximum profit from an item is that the price for the item must satisfy the relationship

$$P_{\text{optimum}} = \frac{nV + P_{\max}}{n + 1} .$$

This relationship requires that price that maximizes profit be related to both the maximum marketable price and the unit variable cost. Price has to do with both affordability of the market segment the business chooses to attract while unit variable-cost has to do with the cost to the business of the item being offered to the market. This finding is not intuitively obvious at first. Typically, pricing consider cost recovery only. We talk of additive-cost pricing, full-cost pricing and pricing at what the market will bear, etc. This pricing formula requires consideration of both the cost of the item to the business and the affordability of the market it serves. For the n = 1 case, the formula speaks to pricing at half what the maximum the market will bear and half of the real (not perceived) cost of the item being sold. In this sense, then, pricing for maximum profit is different in that a judgment or analysis must be made about the values to be used for P_{max} and *n.* Some might argue that doing so is the same as judging what the market will bare. Perhaps true, but experience in pricing finds that few business operators realize the pricing for optimum profit is based on two parameters related to market demand. Again another argument for characterizing the demand model with the relationship

$$N = N_{\max} (1 - P / P_{\max})^n .$$

9 The Theoretical Maximum Profit

What is interesting now is that we can substitute the optimum price into the profit formula and derive a formula for the MAXIMUM THEORETICAL PROFIT (MTP) the item sold can produce. The MTP can then be used as a way to measure the actual performance of a business. The MTP can be the basis of a metric for judging when to change marketing strategies, when to open new stores, when to expand, under what conditions to introduce a new item or start a new business, and how to tune-up an ongoing business to achieve its maximum potential.

Substituting the optimum price formula into the revenue, cost, and profit formulas[13] we can find the MAXIMUM THEORETICAL PROFIT the business can generate as follows:

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[13] This is only possible with mathematics. This is where mathematical tools pay off. The derivation of the optimum pricing formula and the maximum price can only be derived algebraically. The benefit of doing the algebra is that we can study the business by studying the nature of the profit formula. We can learn which variables are of primary importance and which are of secondary importance. When our intuition about business and our analysis of business come together, we

$$\text{Revenue}_{MTP} = \frac{N_{\max}P_{\max}}{4}\left[1-\left(\frac{V}{P_{\max}}\right)^2\right]$$

$$\text{Cost}_{MTP} = \frac{VN_{\max}}{2}\left(1-\frac{V}{P_{\max}}\right)+FT$$

$$\text{Profit}_{MTP} = \frac{N_{\max}P_{\max}}{4}\left(1-\frac{V}{P_{\max}}\right)^2-FT$$

The general MTP takes the form

$$\text{Revenue}_{MTP} = N_{\max}P_{\max}\frac{n^n}{(n+1)^{n+1}}\left(1+\frac{nV}{P_{\max}}\right)\left(1-\frac{V}{P_{\max}}\right)^n$$

$$\text{Cost}_{MTP} = VN_{\max}\left[\frac{n}{(n+1)}\left(1-\frac{V}{P_{\max}}\right)\right]^n+FT$$

$$\text{Profit}_{MTP} = N_{\max}P_{\max}\frac{n^n}{(n+1)^{n+1}}\left(1-\frac{V}{P_{\max}}\right)^{n+1}-FT$$

These are not formulas that guide us on how to make a profit; these are formulas that tell us what the maximum profit and associated revenues and costs will be. Comparing real profitability with this theoretical model will point to areas where the management's understanding of the real market potential in a business either needs improvement or has the satisfaction of knowing the business is appropriately tuned and profitable. Importantly, if the profit from a venture is significantly below it's MTP then fraud, waste or subversion may be the cause. The MTP may be useful in detecting an unlawful cost center. These considerations lead us to another law of business.

10 The Maximum Profit Law

When priced optimally, an item being sold will produce a maximum theoretical profit (MTP) according to the formula

$$\textit{Profit} = N_{\max}P_{\max}\frac{n^n}{(n+1)^{n+1}}\left(1-\frac{V}{P_{\max}}\right)^{n+1}-FT = MTP\ .$$

For the specific case of n = 1

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have the confidence to take bold steps for the business because we have a deep insight into each step. In this sense, we can **implement a bold vision with a conservative strategy**.

$$Profit = \frac{N_{\max} P_{\max}}{4}\left(1-\frac{V}{P\max}\right)^2 - FT = MTP$$

The maximum profit law tells a story. The story is that a business can be tuned for maximum performance. It requires that continuous management attention be given to (1) simultaneous reductions in *V* and FT (2) and increases in P_{max} and N_{max} and (3) maximizing the spread between V and *P*max and (4) working with the customers to maximize their return for continued and loyal business with the firm, *n*. Ted Levit[14] said it the best I have heard: The purpose of marketing is to get and hold customers. To make the business sticky is to make it attractive to customers so that they want to return for continued business.

Of the four terms, the least familiar and has the most leverage is the term $n^n/(n+1)^{n+1}$. Careful analysis of this term shows that as n approaches zero, this term approaches one. Other values lead to prompt reductions in the MTP. For example, when n = 1, the term becomes 0.25 as shown above. Widening the gap between V and Pmax has the next greatest leverage and again is not well known as a factor in business maximizing business performance. It will require careful attention to these two important parameters to achieve peak profit performance. One requires attention to the marketplace and the other with the costs of products and services in the business. Small changes can mean a lot to the MTP of the firm. We will see this later in this paper. Doing so, in conjunction with the *Pricing for Maximum Profit* will assure the business manager that the quantitative aspects of business are not an impediment to business. Note that if the periodic fixed and unit variable costs are near zero, and for *n* near one, the MTP is given by the expressions

$$MTP = \frac{N_{\max} P_{\max}}{4} = \left(\frac{N_{\max}}{2}\right)\left(\frac{P_{\max}}{2}\right).$$

Real fixed costs and real variable costs reduce this profit even more. In words, this relationship says to estimate the maximum possible profit from a business, N_{max} times *P*max and then divide it by four and that is the more likely profit one can expect from the venture. It is the authors' experience that this is where many new businesses, and even some mature businesses introducing a new product or service, set themselves up for very difficult times. When the size of a market is over estimated, and an item is brought to the market priced well below the optimum price, the result can be a very rude awakening followed by a chaotic effort to save the business from failure. Much better to plan the venture pricing at the optimum and forecast the performance of the business using the relationships presented here as the basis for starting a business or introducing a new product.

11 The Law of Two Prices

Returning to our quadratic profit formula, it is important to understand that two prices can result in the same desired profit. If we fix the profit at a desired level P_D, and solve the quadratic formula to determine the price, we find:

[14] Marketing Imagination, Harvard press

$$Price = \frac{P_{max}\left(1 + \frac{V}{P_{max}}\right)}{2} \pm \sqrt{\frac{P^2{}_{max}\left(1 + \frac{V}{P_{max}}\right)^2}{4} - \frac{P_{max}}{N_{max}}\left(P_D + FT + VN_{max}\right)}$$

As mentioned earlier, we see here that there are two prices that will produce the same profit P_D. What is important here is that management must choose the pricing strategy the business should employ for whatever degree of profit success it strives. In its simplest version, the two most well known strategies are to price low and profit by selling a large volume of items or price high and profit by selling a small volume of items. The downside of the former is that it tends to require large inventories and large numbers of low price-driven customers. The up side is that it spreads the risk over a large volume of customers. The latter tends to require smaller inventories and deals with more quality-conscious customers; however, the downside is that the risk of market behavior changes is greater. Both of these strategies lead to sub optimal profits. Interestingly, a combination of dual pricing can lead to above optimum profits[15]. This leads us to yet another law of business:

Except for the singular condition of pricing for maximum profit, there are two prices that will achieve the same level of profit for every item sold. It is management's decision as to which price to employ, considering the market it wants to serve and the style in which it wants the service to be provided. We will see later that to maintain a positive profit flow in businesses where markdowns are made as a function of time, one must price on the high side price. Apart from profit flow, there is not reason to price on either side. Both prices will produce the same level of profit. The matter of pricing must be decided based on the values and purposes of the person owning the business.

With regard to sensitivity, however[16], there is some guidance by noting that the condition for greatest sensitivity of the profit to the price occurs when P is near both V and P_{max}. Counter intuitively for some, if items are priced on the high-side, profits can be INCREASED by reducing the price. If items are priced on the low side, profits drop dramatically with price reductions and can quickly fall to a loss situation. Recovering from a loss position can be extremely difficult if not impossible because the clientele the business has developed is culturally adapted to the low prices of the product. Raising prices can lead to a dramatic loss of customers and their revenues with the result that the business fails. Exactly the opposite is true of its higher priced, quality driven, cousin. Thus we conclude that the greatest threat of loss is met on the low-price side of the "Two Price Law". Said slightly differently, this important finding shows that the closer to a loss (non-profit) a business comes, the greater the sensitivity to price and eventual failure. If you have a choice, be on the high side as recovery is possible with price reductions, a more palatable option in the marketplace. This relationship argues for not operating a business on the low-side price, but rather to operate on the high-side price.

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[15] This explains why a high-end retail store can be operated near its low-end outlet store, and the combination of the profits from the two stores will exceed the profits from either store operating at peak profits alone.

[16] See more on sensitivity near the end of this paper.

12 Profit Flow

Profit flow is denned to be the rate of change, in time *t*, of profit. In mathematical terms, it is the derivative with respect to time of the profit formula. Generally, profit varies with time in two ways. The first is the profit generated by sales. The second is related to the reduction in price over time *t* to move product. Seasonal reductions in price to move fashionable products are an example of the second type of effect on profit. Let us assume that these price reductions are linear in time[17]. Thus, we would expect that the price reduction relationship would have the form,

$$P = P_s\left(1 - \frac{t}{\tau}\right)$$

Here, (*t* is time and τ is the maximum time over which the price reductions take place. Note that,

$$\frac{\partial P}{\partial t} = -\frac{P_s}{\tau}$$

Since *P*, and τ are positive, we see the partial is negative, an important point we shall see soon. Now let us write the profit flow as follows:

$$\frac{\partial profit}{\partial t} = \frac{\partial P}{\partial t}\frac{N_{\max}}{P_{\max}}\left(P_{\max} + V - 2P\right) - F$$

First note that if $\frac{\partial profit}{\partial t}$ is to be greater than zero, the terms in the parentheses must be greater than zero and this is only possible if

$$P \geq \frac{P_{\max} + V}{2}$$

Said in more interesting terms, the price must be greater than the optimum price. This argues for pricing on the high side of the optimum, an issue we considered earlier in this paper.

Second, we see that for $\frac{\partial profit}{\partial t}$ to always be positive, the first term in the derivative must be greater than the total fixed cost. Solving for the price that will meet this requirement we find

[17] like price elasticity, price reductions are generally nonlinear, occurring in discrete price reductions. In our analysis here, we will assume that there is a line that, on the average, passes through the "stairstep" of discrete price reductions. This simplification is valid in the math modeling of business because our purpose is to understand the effect of price reductions on the laws of business, not for the precision of the spreadsheet analysis that is required to consider these price reductions for detailed business analysis.

$$P \geq \frac{1}{2}\left(P_{max} + V - \frac{\frac{\partial P}{\partial t} FN_{max}}{P_{max}}\right)$$

Remembering that $\frac{\partial P}{\partial t}$ in negative we see than that the price to set depends on all the management variables we have discussed before, and in the context of maintaining a price beyond the optimum. This analysis shows that for those products that involve regular price reductions, pricing beyond the optimum is required if a positive profit flow is to be retained for all time. The problem with this type of pricing is that the business does not realize a maximum possible profit. If, however, a profit flow that is greater than, or equal to zero "on the average"[18], then the price that will result in achieving the maximum possible profit is

$$P \geq \frac{1}{2}\left(P_{max} + V\right) - \frac{\frac{\partial P}{\partial t} FN_{max}}{4P_{max}}$$

These pricing considerations can be captured in the following rule of business.

13 The Profit Flow Rule

For products and/or services whose prices decrease in time, the price that will maximize profit on the average is given by the relation:

$$P \geq \frac{1}{2}\left(P_{max} + V\right) - \frac{\frac{\partial P}{\partial t} FN_{max}}{4P_{max}} .$$

14 Sensitivities

For the sake of completeness, the sensitivities of the profit formula are provided for those who will be interested in ranking the variables to change when choosing a profit improvement strategy. Differentiating the profit formula with respect to each variable gives us the important information about the sensitivity of the profits to each of the five management variables. The sensitivity formulas for the case *n* = 1 are presented in the following array:

$$\frac{\partial profit}{\partial P} = N_{max} \frac{\left(P_{max} + V - 2P\right)}{P_{max}}$$

[18] Allowing first a positive profit flow and then a negative flow, which on the average is zero or greater

$$\frac{\partial profit}{\partial V} = -N_{\max} \frac{(P_{\max} - P)}{P_{\max}}$$

$$\frac{\partial profit}{\partial P_{\max}} = N_{\max} \frac{P(P - V)}{P^2{}_{\max}}$$

$$\frac{\partial profit}{\partial N_{\max}} = \frac{(P - V)(P_{\max} - P)}{P_{\max}}$$

$$\frac{\partial profit}{\partial F} = -T$$

Note that when we set the sensitivity (slope) of the profit with respect to the price equal to zero, the condition that occurs when the profit is a maximum, we find the optimum pricing formula as follows:

$$\frac{\partial profit}{\partial P} = N_{\max} \frac{(P_{\max} + V - 2P)}{P_{\max}} = 0$$

when the parenthetical term also equals zero as,

$$P_{max}+V-2P=0$$

we can see that the price that maximizes the profit function is

$$P = \frac{(P_{\max} + V)}{2}$$

A similar array of sensitivities applies for the case where the price is at the optimum and the profits are at a maximum, except in the case for price, where the sensitivity of the optimized profit function is zero. These sensitivities are also presented here for the sake of completeness. The sensitivity formulas for the profit at the optimum price are:

$$\frac{\partial profit_{\max}}{\partial P} = 0$$

$$\frac{\partial profit_{\max}}{\partial V} = \frac{-N_{\max} V}{2P_{\max}}$$

$$\frac{\partial profit_{\max}}{\partial P_{\max}} = N_{\max} \left(\frac{P^2{}_{\max} - V^2}{4P^2_{\max}} \right)$$

$$\frac{\partial profit_{\max}}{\partial N_{\max}} = \frac{P^2{}_{\max} - V^2}{4P_{\max}}$$

$$\frac{\partial profit_{\max}}{\partial F} = -T$$

These sensitivities can be used to analytically determine the leverage each variable has on the profit of an on-going business in the case of the first array and in the case of the MTP in the case of the second array.

15 Summaries

The purpose of this research is to help start-up and small community businesses ensure success, by focusing on profit. The findings of this research are that there are general laws and rules that apply to business. The laws relate profit to the five fundamental business variables of price, the fixed and variable costs of doing business, the maximum price the market will bear for the product being sold and the maximum size of the market. These laws and rules take into account (1) the economic law of price elasticity in the market and (2) the business practice of reducing prices over time to move seasonal products. A careful analysis of the laws of business lead to the identification of rules for successful business operations. Businesses that fail violate these rules in one-way or another. To avoid failure and ensure success, management need only apply the rules to their business. Also, presented are the sensitivities of profit to the fundamental variables of business in general and pricing for maximum profit in particular. These sensitivities are useful for determining the order of importance of changes to be made to improve business performance.